Applied Multilevel Regression Models

The advantages of using hierarchical models were (i) the ability to measure hierarchical data, (ii) the adaptation to independence of observation, (iii) the adaptation to a potential homoscedasticity (variances of regions most likely are unequal), and (iv) analysis of data where all units do not have the same number of data points across time (Goldstein 1995).

Prior to performing these analyses, the time variable was rescaled to make the year 1950, when about half of the countries existed as sovereign states (87 countries out of the total 177), and the centered time is *1950=0* rather than 1800, the initial year of this study, when only 16 countries were independent. Hence, the intercept in the growth model was specified to represent the average status of world democratization in 1950 to allow for a more meaningful interpretation of the results. Moreover, the parameter of a regression line for an average country in the world and in each region (the rate of democratic growth) was estimated and slopes were selected as a year squared.[11]

In addition, valid time spans were identified. Two main criteria were employed in selecting these time spans: (a) the existence of sufficient variability in democratic level; and (b) the number of sovereign states as a percentage of the total number of states existing at the end of the 1990s. The years prior to the time when sufficient variability in the average democratization level was observed and the years prior to when at least one-fourth of the states in the world or in a particular region received independence, were omitted.

Accordingly, for the world analysis, the years 1860– 1999 were selected; for the Americas, 1820–1999; for Europe, 1800–1999; for Africa: 1949–1999; for the Middle East: 1917–1999; and for Asia: 1900–1999. Oceania, which includes a small number of countries and no variability in democratization (only four states in the region are democratic, two of which were highly democratic for over a century), was excluded from the regional analyses.

In studies on democracy, I apply the two-level growth models for the regional analyses, and the three-level growth model for the world analysis. In addition to evaluating the means for the intercept and the slope as a function of variables at the within-country level (country-year level covariates), variables that are characteristic to each country (country-level covariates) and/or each region (region-level covariates), the variances (marked differences) are estimated. The variances in the level and the rate of democratic growth among regions; the variance for the intercepts, the slopes, and the covariance for intercepts and slopes between countries; and the residual are calculated.

The modeling was implemented in "SAS PROC MIXED," a procedure that allows hierarchical modeling (Singer 1998; Singer and Willett 2003). In the simplest form (without added predictors), the two-level growth model is expressed as the sum of two parts: a fixed part with two fixed effects (for the intercepts and for the effect of time) and a random part, which contains estimates of variances for the intercepts, the time slope, the covariance, and the residual. The following equation summarizes the simplest form of the two-level hierarchical growth model employed:

$$Y_{ij} = (\beta_{00} + \beta_{10} Year_{ij}) + (e_{oj} + e_{1j} Year_{ij} + r_{ij}),$$

where $e_{oj} \sim N(0, \tau_{00})$ $e_{1j} \sim N(0, \tau_{10})$ $r_{ij} \sim N(0, \sigma^2)$

β_{00} represents an average democracy level in the world in 1950

$\beta_{10}Year_{ij}$	represents an average slope of democratic growth in the world
τ_{00}	represents variation in the democracy level among countries (between countries' intercepts)
τ_{10}	represents variation among countries' temporal rate of democratic growth (among countries' slopes)
σ^2	represents residual (the within country variance)
ij	the subscript denotes the within country level (the level-1)
j	the subscript denotes the country or region level (level-2)

The dependent variable Y (growth of democracy) is explained with an intercept β_{00} and a slope $\beta_{10}Year_{ij}$. The fixed part of the model contains fixed effects for the intercept (β_{00}) and for the effect of time ($\beta_{10}Year_{ij}$). The random part contains three estimates of variances: for the intercept (e_{oj}), representing variation in the democracy level between countries in the world; for the slope of time ($e_{1j}\ Year_{ij}$), representing variation in the slope of the temporal rate of democratic growth among countries in the world; and for the within-country residual (r_{ij}), representing variation in democracy level within countries or the departure from the predicted score of the country's actual score in 1950 on the democracy scale.

To explore whether the variations within a country relates to covariates, or whether intercepts and slopes are related to covariates, independent variables were added to this unconditional model, where a variable X_{ij} represents the level-1 (within-country) predictor and a higher-level variable X_j , the level-2 (country- or region-level) predictor. For example, a model with a level-1 and level-2 covariate is summarized by the following equation:

$$Yij = [\beta_{00} + \beta_{10}Year_{ij} + \beta_{01}X_{ij} + \beta_{11}(X_{ij})(Year_{ij}) + \beta_{01}X_j + \beta_{11}(X_j)(Year_{ij})] + [e_{oj} + e_{1j}(Year_{ij}) + r_{ij}]$$

This model is also expressed as the sum of two parts of fixed and random effects. The fixed effects for the intercept (β_{00}) represent an average democracy level in the world in 1950; for the slope, ($\beta_{10}Year_{ij}$) represents the average growth rate of democracy; the predictor of democracy level $\beta_{01}X_{ij}$ and $\beta_{01}X_j$ captures the relationship between the covariates and the status of democracy in 1950; and the predictor of slope $\beta_{11}X_{ij}(Year_{ij})$ and $\beta_{11}X_j$ ($Year_{ij}$) captures the relationship between the covariates and the growth rate of democracy. As in the unconditional model, the random part represents a variation in the intercepts, the slopes, and the residual. With the variance in democratic level and the rate of democratic growth among regions added to this model, the above two-level model became a three-level growth model to which more covariates were added .

To allow the intercept and the slope to vary across countries and across regions, a structure of the variance-covariance was selected using the goodness-of-fit statistics, and the UN (unstructured) structure was indicated as best fitting the data (Singer and Willett 2003). The UN model was selected out of the tested CS, CSH, HF, ARH, AR, and UN models, using goodness-of-fit statistics from Akaike's Information Criterion (AIC); Akaike's Information Criterion corrected for sample size (AICC); Bayesian Information Criterion (BIC); and a subsequent Likelihood ratio test (–2 RLL). The UN structure indicates that the model does not place any structure on the variance for intercepts and variance for slopes, nor is any structure imposed on the covariance between these two.

Comparison of the results of the UN model with the simple model, which did not impose additional structure on the error covariance matrix (beyond the heteroscedastic structure of the intercept and slopes as outcome models), indicated that, once the covariance of the intercepts and slopes had been introduced, no additional autoregressive error structure needed to be added. Nonetheless, one more test was performed, with results leading to the same conclusion. One of the strengths of PROC MIXED is that it allows adding complexity to test for autoregressive error structure, once the covariate has been taken into account, and to compare different structures for the error covariance matrix. In this study, following Singer (1998), the extra complexity was added to the assessment of the covariate estimate for the autoregressive parameter, and fit

statistics pointed toward the conclusion that no additional complexity of the autoregressive error structure needed to be added to the final model.

Since this methodology has not been used in previous research on democracy, I derived slopes that best fit the observed data and, using the depicted time spans, assessed the validity of the unconditional growth models (the three-level for the world and the two-level for each region) by evaluating the estimated rate and level of democratic growth in the world and in each world's region against observed data. The slopes that best fit the observed data were depicted as year squared for the world and the Americas, year for Europe, and cubic year for Africa, the Middle East, and Asia. Note that these slopes are not identical with the slope of a year squared selected for the world and all regional analyses. As I estimated the year squared slopes were the best that fit the observed data not only across the world but also across regions; and the slopes that best fit the observed data varied between a year squared in the Americas, a year in Europe, and a year cubed in Africa, the Middle East, and Asia. However, to prioritize logical consistency over a model that fits the data best, a year squared was selected for all analyses.